Problema 2. Arătaţi că numărul $2^{2015} + 4^{n^2+n} + 9^{n^2+3n+1}$ nu este pătrat perfect, oricare ar fi n număr natural.

* * *

$U(2^{2015}) = U(2^{4\cdot 503+3}) = 8$

$n^2+n = n\cdot(n+1)$

I $n =$ par, $n+1 =$ impar $\} \Rightarrow n\cdot(n+1) =$ par

II $n =$ impar, $n+1 =$ par $\} \Rightarrow n\cdot(n+1) =$ par $\} \; n(n+1) =$ par

1. $n \neq 0 \Rightarrow n\cdot(n+1) =$ par (nenul)

$U(4^{n^2+n}) = 6$

$n^2+3n+1 = n^2+n+2n+1 = \underbrace{\underbrace{n\cdot(n+1)}_{\text{par}} + \underbrace{2n+1}_{\text{impar}}}_{\text{impar}}$

$U(9^{n^2+3n+1}) = 9 \Rightarrow U(2^{2015} + 4^{n3+3} + 9^{2n+3n+1}) = U(8+6+9) = U(23) = 3 \Rightarrow 2^{2015} + 9^{2n+3n^2+1} + 4^{n^2+n} \neq$ p.p.

II $n = 0 \Rightarrow 2^{2015} + 4^{n^2+n} + 9^{n^2+3n+1} = 2^{2015} + 4^0 + 9^1 = 2^{2015} + 10 = U(2^{2015}+10) = U(8+0) = 8 \Rightarrow$

$\Rightarrow 2^{2015} + 10 \neq$ p.p.