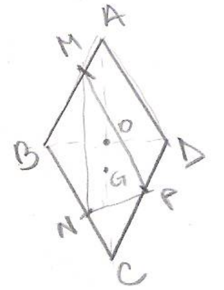
2.

Fie rombul $ABCD$ şi punctele $M \in (AB), N \in (BC), P \in (CD)$. Să se arate că centrul de greutate al triunghiului MNP aparţine dreptei AC dacă şi numai dacă $AM + DP = BN$.



Fie G centrul de greutate al ΔMNP

$\Rightarrow \vec{GM} + \vec{GN} + \vec{GP} = \vec{0}$

$\vec{GA} + \vec{AM} + \vec{GB} + \vec{BN} + \vec{GD} + \vec{DP} = \vec{0}$

fie $AC \cap BD = \{O\} \Rightarrow \vec{GB} + \vec{GD} = 2\vec{GO} = \vec{GA} + \vec{GC}$ (O mijl BD, AC)

$\Rightarrow 2\vec{GA} + \vec{GC} + \vec{AB} \cdot \frac{AM}{AB} + \vec{BC} \cdot \frac{BN}{BC} + \vec{DC} \cdot \frac{DP}{DC} = \vec{0}$ $(\vec{GC} = \vec{GA} + \vec{AC})$

$\xRightarrow{\vec{AB} = \vec{DC}} 3\vec{GA} + \vec{AC} + \vec{AB} \cdot \frac{AM + PD}{AB} + \vec{BC} \frac{BN}{AB} = \vec{0}$

I dacă $G \in AC$

$\Rightarrow 3\vec{GA} + \vec{AC} = \vec{AC} \cdot \alpha$ $(\vec{GA}, \vec{AC}$ coliniari$)$

$\Rightarrow \vec{AC}\alpha = (\vec{AB} + \vec{AD})\alpha = \vec{AB}\alpha + \vec{BC} \cdot \alpha$

$\Rightarrow \vec{AB}\left(\alpha + \frac{AM + PD}{AB}\right) + \vec{BC}\left(\alpha + \frac{BN}{AB}\right) = \vec{0}$

dar $\vec{AB}, \vec{AC}$ nu sunt coliniari $\Rightarrow \alpha + \frac{AM + PD}{AB} = \alpha + \frac{BN}{AB} = 0$

$\Rightarrow \underline{AM + PD = BN}$

II dacă $AM + PD = BN$

fie $\lambda = \frac{AM + PD}{AB} = \frac{BN}{AB}$

$\xRightarrow{\vec{AB} + \vec{AD} = \vec{AC}} 3\vec{GA} + \vec{AC} + \vec{AB} \cdot \lambda + \vec{BC} \cdot \lambda = 3\vec{GA} + \vec{AC} + (\vec{AB} + \vec{AD})\lambda =$

$3\vec{GA} + \vec{AC} \cdot (\lambda + 1) = \vec{0}$

$3 \neq 0$; $\lambda + 1 \neq 0$ $\Rightarrow \vec{GA}, \vec{AC}$ coliniari

$\Rightarrow GA \parallel AC \Rightarrow A, G, C$ coliniare $\Rightarrow \underline{G \in AC}$

$\xRightarrow{I; II}$ $\underline{G \in AC \Leftrightarrow AM + PD = BN}$

q.e.d.