

**Problema 3.** Determinați numerele naturale  $a, b, c$  știind că

$$\frac{a}{3} = \frac{b}{4} = \frac{4c+9}{3c-2}$$

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Rezolvare:

$$\text{Din } \frac{a}{3} = \frac{4c+9}{3c-2} \text{ rezultă: } a(3c-2) = 12c+27 \Rightarrow$$

$$\Rightarrow 3ac - 2a = 12c + 27 \Rightarrow 3ac - 12c = 2a + 27 \Rightarrow$$

$$\Rightarrow 3c(a-4) = 2a + 27 \Rightarrow c = \frac{2a+27}{3(a-4)}$$

$$\text{Cum } c \in \mathbb{N} \Rightarrow a-4 \mid 2a+27$$

$$a-4 \mid 2a-8$$


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$$a-4 \mid 35 \Rightarrow$$

$$\Rightarrow a-4 \in \{1, 5, 7, 35\} \Rightarrow$$

$$\Rightarrow a \in \{5, 9, 11, 39\}$$

$$1. a=5 \Rightarrow c = \frac{37}{3} \notin \mathbb{N}.$$

$$2. a=9 \Rightarrow c = \frac{48+27}{15} = \frac{75}{15} = 3 \Rightarrow c=3;$$

$$\text{Din } 4a = 3b \Rightarrow 3b = 36 \Rightarrow b = 12;$$

$$\text{Cum } \frac{9}{3} = \frac{12}{4} = \frac{42+9}{9-2} = 3 \Rightarrow (a, b, c) = (9, 12, 3) = \text{SOLUȚIE}$$

$$3. a=11 \Rightarrow c = \frac{22+27}{3 \cdot 7} = \frac{49}{21} = \frac{7}{3} \notin \mathbb{N}$$

$$4. a=39 \Rightarrow c = \frac{78+27}{3 \cdot 35} = \frac{105}{105} = 1 \Rightarrow c=1;$$

$$\text{Din } 4a=3b \Rightarrow 3b=4 \cdot 39 \Rightarrow b=4 \cdot 13=52$$

$$\text{Cum } \frac{39}{3} = \frac{52}{4} = \frac{4+9}{3-2} = 13 \Rightarrow$$

$$\Rightarrow (a, b, c) = (39, 52, 1) = \text{SOLUȚIE}$$

Răspuns:

$$\{(a, b, c) = (9, 12, 3); (39, 52, 1)\}$$