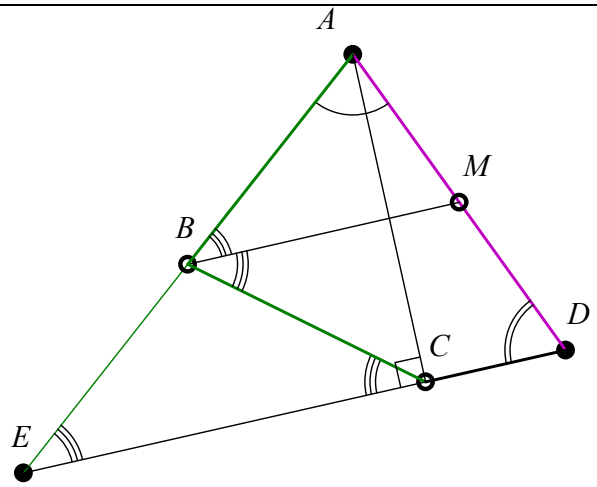
Problema 3.

Problema 5502 din Matematika v Skole, nr.3/2017; p.73:

În patrulaterul convex $ABCD$, $m\left(\widehat{BCD}\right)=m\left(\widehat{BAD}\right)+m\left(\widehat{ADC}\right)$ **şi bisectoarea unghiului** $\widehat{ABC}$ **trece prin mijlocul** M **– al laturii** $[CD]$. **Găsiţi** $m\left(\widehat{ACD}\right)$!



SOLUŢIE (*Mihai Miculiţa*): Notând cu $\{E\}=AB\cap CD$, din ΔAED, obţinem că:

$$\left.\begin{array}{r} m\left(\widehat{BEC}\right)=180^0-\left[m\left(\widehat{BAD}\right)+m\left(\widehat{ADC}\right)\right] \\ m\left(\widehat{BCD}\right)=m\left(\widehat{BAD}\right)+m\left(\widehat{ADC}\right);(ip) \end{array}\right\}\Rightarrow \boxed{m\left(\widehat{BEC}\right)}=180^0-m\left(\widehat{BCD}\right)=\boxed{m\left(\widehat{ECB}\right)};(1)\Rightarrow$$

$$\Rightarrow [BE]\equiv[BC]. \quad (2)$$

Din triunghiul isoscel BEC, obţinem acum că:

$$\left.\begin{array}{r} \left.\begin{array}{r} m\left(\widehat{ABC}\right)=m\left(\widehat{BEC}\right)+m\left(\widehat{ECB}\right) \\ m\left(\widehat{BEC}\right)=m\left(\widehat{ECB}\right);(1) \end{array}\right\}\Rightarrow m\left(\widehat{ABC}\right)=2\cdot m\left(\widehat{BEC}\right) \\ \widehat{ABM}\equiv\widehat{MBC};(ip)\Rightarrow m\left(\widehat{ABC}\right)=2\cdot m\left(\widehat{ABM}\right) \end{array}\right\}\Rightarrow \widehat{BEC}\equiv\widehat{ABM}\Rightarrow BM\parallel DE$$

$$[AM]\equiv[MD];(ip)\Bigg\}\Rightarrow$$

$$\left.\begin{array}{r} \Rightarrow [AB]\equiv[BE] \\ [BE]\equiv[BC];(2) \end{array}\right\}\Rightarrow [AB]\equiv[BE]\equiv[BC]\Rightarrow AC\perp CD\Rightarrow \boxed{m\left(\widehat{ACD}\right)=90^0}.\ \blacksquare$$

Observaţie: Dacă într-un triunghi ascuţitunghic ADE, **ducem înălţimea** AC **şi notăm cu** B **– mijlocul laturii** $[AE]$, **patrulaterul** $ABCD$ **astfel obţinut are toate proprietăţile din ipoteza problemei 5502.**