**P3.**a) Fie $n \in \mathbb{N}^*$. Arătaţi că numerele $x_k = ctg^2 \frac{k\pi}{2n+1}$, $k = \overline{1, n}$ sunt soluţiile ecuaţiei:

$$C_{2n+1}^1 x^n - C_{2n+1}^3 x^{n-1} + C_{2n+1}^5 x^{n-2} - \ldots + (-1)^k C_{2n+1}^{2k+1} x^{n-k} + \ldots + (-1)^n = 0.$$

b) Calculaţi sumele

$$S_1(n) = \sum_{k=1}^{n} ctg^2 \frac{k\pi}{2n+1}, \quad S_2(n) = \sum_{k=1}^{n} ctg^4 \frac{k\pi}{2n+1}, \quad S_3(n) = \sum_{k=1}^{n} ctg^6 \frac{k\pi}{2n+1}.$$

c) Arătaţi că

$$ctg^2 x < \frac{1}{x^2} < ctg^2 x + 1, \ (\forall) x \in \left(0, \frac{\pi}{2}\right).$$

d) Calculaţi limitele

$$\lim_{n \longrightarrow \infty} \sum_{k=1}^{n} \frac{1}{k^2}, \quad \lim_{n \longrightarrow \infty} \sum_{k=1}^{n} \frac{1}{k^4}, \quad \lim_{n \longrightarrow \infty} \sum_{k=1}^{n} \frac{1}{k^6}.$$