



Problema 3. Fie p, q, r numere naturale prime distincte. Determinați valorile lui p, q, r pentru care $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr}$ este număr natural.

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$p, q, r =$ prime distincte

Relatia este simetrică, deci putem considera:

$$p < q < r \Rightarrow p \geq 2, q \geq 3, r \geq 5$$

$$\text{Dacă } p \neq 2 \Rightarrow p \geq 3, q \geq 5, r \geq 7 \Rightarrow$$

$$\Rightarrow \frac{1}{p} \leq \frac{1}{3}; \frac{1}{q} \leq \frac{1}{5}; \frac{1}{r} \leq \frac{1}{7} \Rightarrow$$

$$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} \leq \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{71}{105} < 1 \Rightarrow$$

$$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr} < \frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1 \Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr} \notin \mathbb{N},$$

fals $\Rightarrow p = 2$

$$\text{Dacă } q \neq 3 \Rightarrow q \geq 5, r \geq 7 \Rightarrow$$

$$\Rightarrow \frac{1}{q} \leq \frac{1}{5}; \frac{1}{r} \leq \frac{1}{7} \Rightarrow$$

$$\Rightarrow \frac{1}{q} + \frac{1}{r} \leq \frac{1}{5} + \frac{1}{7} \Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} \leq \frac{35}{2} + \frac{14}{5} + \frac{10}{7} = \frac{59}{70} < 1 \Rightarrow$$

$$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr} < \frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1 \Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr} \notin \mathbb{N}, \text{ fals} \Rightarrow$$

$$\Rightarrow q = 3$$

$$\text{Dacă } r \neq 5 \Rightarrow r \geq 7 \Rightarrow$$

$$\Rightarrow \frac{1}{r} \leq \frac{1}{7} \Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} \leq \frac{21}{2} + \frac{14}{3} + \frac{6}{7} = \frac{41}{42} < 1 \Rightarrow$$

$$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr} < \frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1 \Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr} \notin \mathbb{N}, \text{ fals} \Rightarrow$$

$$\Rightarrow r = 5$$

Soluție: $p = 2$
 $q = 3$
 $r = 5$

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