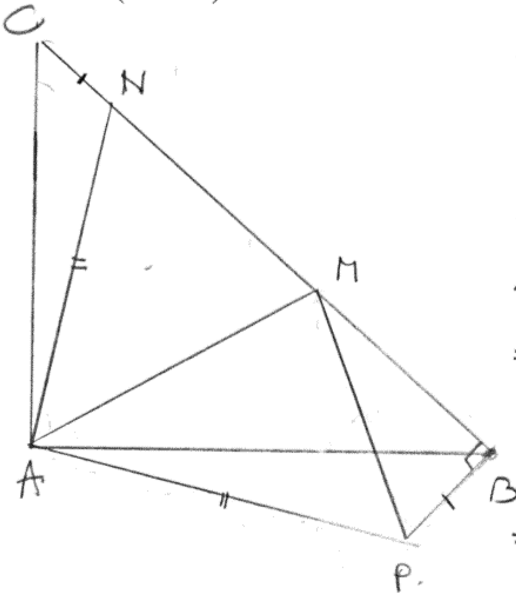
Fie triunghiul dreptunghic isoscel ABC cu $m(\sphericalangle BAC)=90^\circ$ şi punctele $M, N \in (BC)$ astfel încât $m(\sphericalangle MAN)=45^\circ$. Să se demonstreze că $BM^2+CN^2=MN^2$.



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Fie P în semiplanul opus determinat de AB față de punctul C, astfel încât $\triangle NAP$ dreptunghic isoscel.

$$\left.\begin{array}{l} \Rightarrow \sphericalangle NAP = 90^\circ \\ \text{dar } \sphericalangle BAC = 90^\circ \end{array}\right\} \Rightarrow \sphericalangle CAN \equiv \sphericalangle PAB$$

$$\underset{(LUL)}{\Rightarrow \triangle CAN \equiv \triangle BAP} \left\{\begin{array}{l} \sphericalangle CAN \equiv \sphericalangle PAB \\ AB \equiv AC \\ AP \equiv AN \end{array}\right.$$

$$\left.\begin{array}{l} \Rightarrow \left\{\begin{array}{l} \sphericalangle ABP = 45^\circ \Rightarrow \sphericalangle MBP = 90^\circ \Rightarrow (\text{cf teoremei lui Pitagora}) \\ CN \equiv PB \end{array}\right. \\ MP^2 = MB^2 + BP^2 \end{array}\right\} \Rightarrow MP^2 = MB^2 + NC^2 \ (1)$$

dar $\sphericalangle MAN = 45^\circ \Rightarrow \sphericalangle MAP = 45^\circ \Rightarrow$

$$\left.\begin{array}{l} \underset{(LUL)}{\Rightarrow \triangle NAM \equiv \triangle PAM} \Rightarrow MN \equiv MP \\ (1) \end{array}\right\} \Rightarrow \boxed{MN^2 = MB^2 + NC^2}$$