SOLUȚIE

Problema 1

Arătaţi că pentru orice $a, b, c, d > 0$ are loc inegalitatea

$$\frac{1}{a} + \frac{2}{\sqrt{b}} + \frac{3}{\sqrt[3]{c}} + \frac{4}{\sqrt[4]{d}} \geqslant \frac{10}{\sqrt[10]{abcd}}.$$

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Soluţie. Folosind inegalitatea mediilor avem: $\frac{1}{a} + \frac{2}{\sqrt{b}} + \frac{3}{\sqrt[3]{c}} + \frac{4}{\sqrt[4]{d}} =$

$$= \frac{1}{a} + \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{b}} + \frac{1}{\sqrt[3]{c}} + \frac{1}{\sqrt[3]{c}} + \frac{1}{\sqrt[3]{c}} + \frac{1}{\sqrt[4]{d}} + \frac{1}{\sqrt[4]{d}} + \frac{1}{\sqrt[4]{d}} + \frac{1}{\sqrt[4]{d}}$$

$$\geqslant 10 \sqrt[10]{\frac{1}{a} \cdot \frac{1}{\sqrt{b}} \cdot \frac{1}{\sqrt{b}} \cdot \frac{1}{\sqrt[3]{c}} \cdot \frac{1}{\sqrt[3]{c}} \cdot \frac{1}{\sqrt[3]{c}} \cdot \frac{1}{\sqrt[4]{d}} \cdot \frac{1}{\sqrt[4]{d}} \cdot \frac{1}{\sqrt[4]{d}} \cdot \frac{1}{\sqrt[4]{d}}}$$

$$= 10 \cdot \frac{1}{\sqrt[10]{abcd}}.$$